MATH 105 101 Midterm 1 Sample 1

1. (20 marks)

   (a) (5 marks) Let

   $$f(x, y) = \arctan(xy)$$

   Compute $f_{xx}(1, 1)$ and $f_{xy}(1, 1)$. *Simplify your answer.*

   (b) (5 marks) Find <u>all</u> vectors in $\mathbb{R}^3$ of length 10 which are parallel to the vector $\langle 3, 0, -4\rangle$.

   (c) (2 marks) Find an equation of the plane $\mathcal{P}$ passing through the point $(3, -1, 4)$ that is parallel to the plane $3x - 5y = 3$.

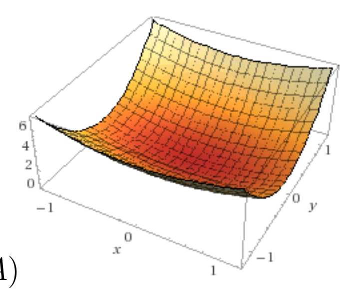
   (d) (3 marks) Is there a function $f(x, y)$ such that $\nabla f(x, y) = \langle \cos y, x \sin y + y^2\rangle$? If not, explain why no such function exists; otherwise find $f(x, y)$. State clearly any result that you use.

   (e) (5 marks) Consider the surface S given by:

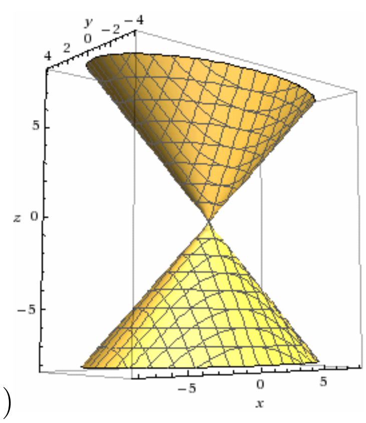
   $$z^2 = x^2 + 4y^2$$

   (i) (4 marks) Sketch the traces of S in the $z = 1$ and $y = 0$ planes.

   (ii) (1 mark) Based on the traces you sketched above, which of the following renderings represents the graph of the surface?



(A)



(B)

2. (10 marks) Let R be the semicircular region $\{x^2 + y^2 \le 4, x \le 0\}$. Find the maximum and minimum values of the function

   $$f(x, y) = x^2 + y^2 + 2x$$

   on the *boundary of the region R*.

3. (10 marks) Find *all* critical points of the following function:

   $$f(x, y) = x^2y - 2xy^2 + 3xy + 4$$

   Classify each point as a local minimum, local maximum, or saddle point.

4. (10 marks) A consumer has the following utility function for the two types of chocolates:

$$f(x, y) = 5x^2 + 6xy + y^2 + 38x + 18y$$

where x and y represent the number of grams of milk chocolate and dark chocolate, respectively. Suppose that a gram of milk chocolate costs \$10 and a gram of dark chocolate costs \$5, and the consumer can spend \$40 on the two types of chocolates. Use Lagrange multipliers to find how much of each type of chocolates the consumer should buy to maximize his utility value. Clearly state the objective function and the constraint. *You are not required to justify that the solution you obtained is the absolute maximum.* **A solution that does not use the method of Lagrange multipliers will receive no credit, even if it is correct.**